Formal Methods & Tools

University of Twente
Enschede - The Netherlands

Actual Test Coverage for Embedded Systems

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University of Twente

14th Dutch Testing Day
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Contents

Introduction – coverage

Why coverage?

- Testing is inherently incomplete
- Testing does increase our confidence in the system
- A notion of *quality* of a test suite is necessary
- Coverage: ‘amount’ of specification / implementation examined by a test suite

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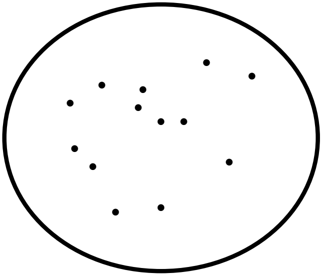

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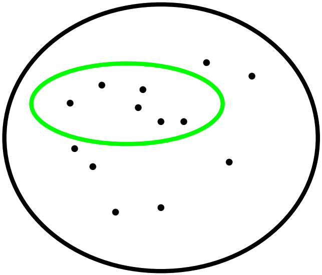

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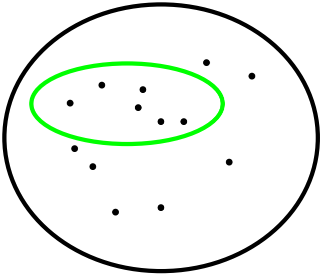

Coverage: $\frac{6}{13} = 46\%$.

Early work on coverage

- Statement coverage
- Path coverage

Introduction – Existing approaches to coverage

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Limitations: - all faults are considered of equal severity
- no probabilities
- syntactic point of view

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Quality: $\frac{4}{6} \cdot 10 = 6.7$

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Starting point for my work: semantic coverage

Previous work by Brandán Briones, Brinksma and Stoelinga

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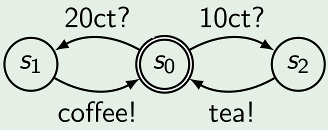

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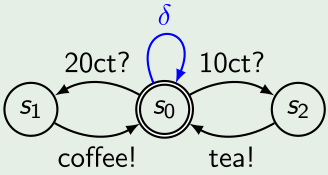

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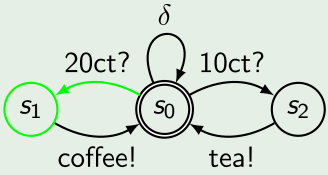

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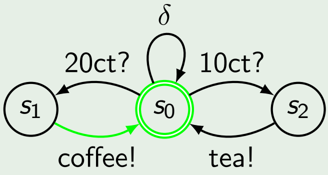

20ct? coffee!

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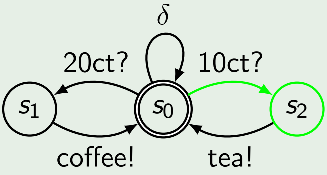

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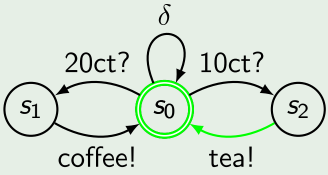

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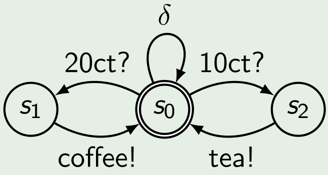

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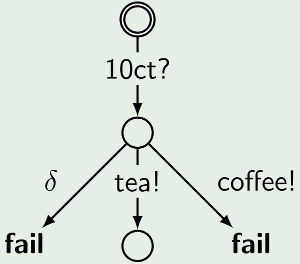
Test cases



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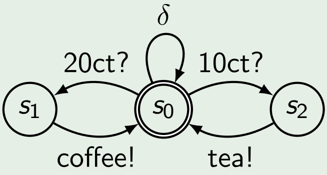

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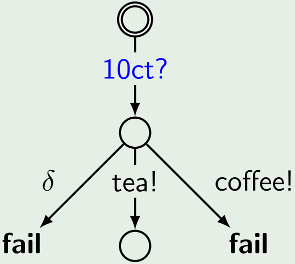

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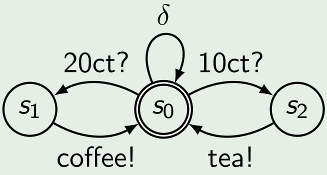

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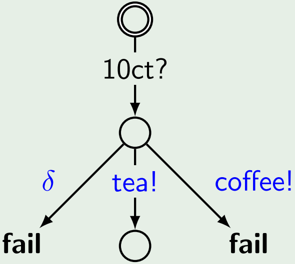

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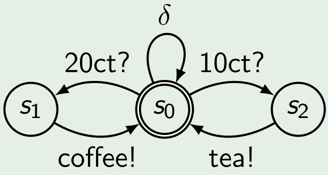

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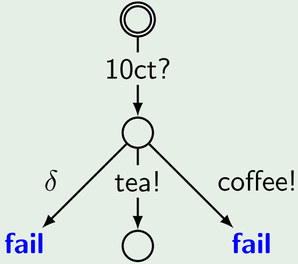

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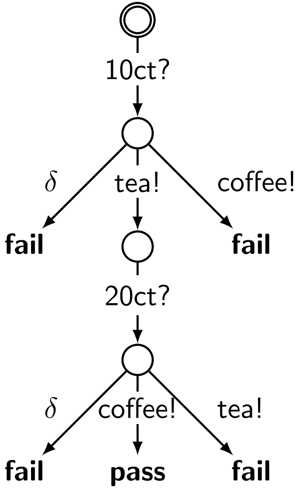

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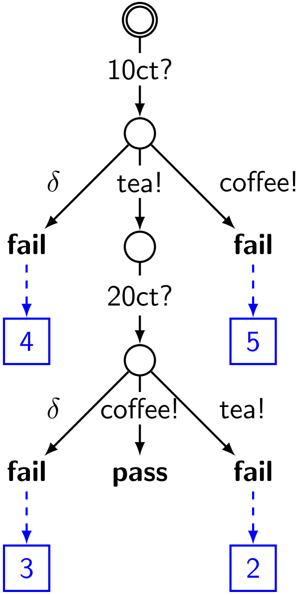

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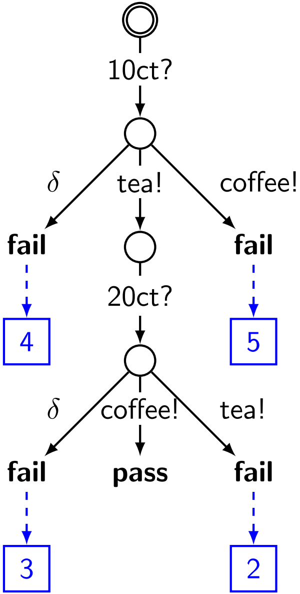
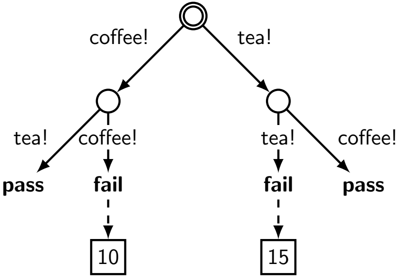
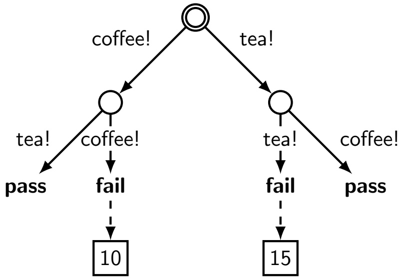


Absolute potential coverage

$4 + 5 + 3 + 2 = 14$

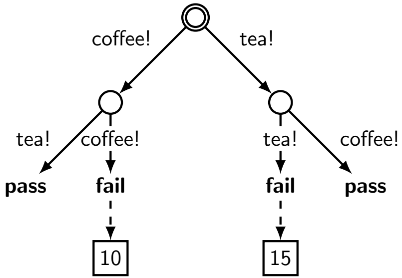
Introduction - Limitations of potential coverage





Absolute potential coverage

$$10 + 15 = 25$$



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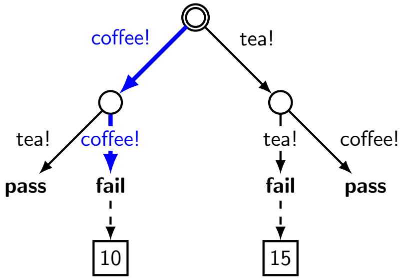
Some remarks

- Not all faults can be detected at once
- Single executions cover only some faults
- Executing more often could increase coverage
- How many executions are needed?
- Necessary to include probabilities!



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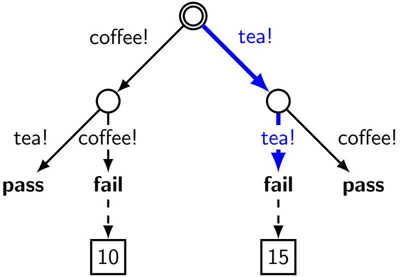

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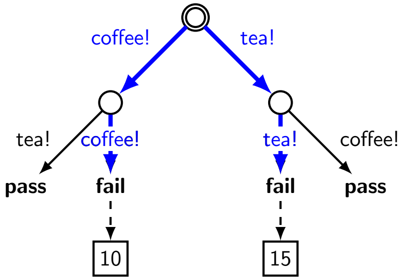

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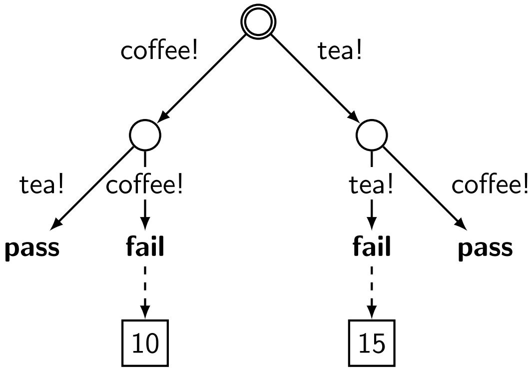
3. Predicting actual coverage:

   Predicting the actual coverage a test case or test suite yields.

Fault coverage



Fault coverage

A fault is *covered* by an execution if the execution gives us information about whether the fault is present or absent

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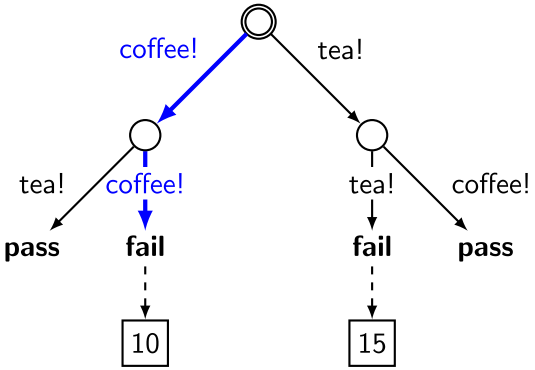

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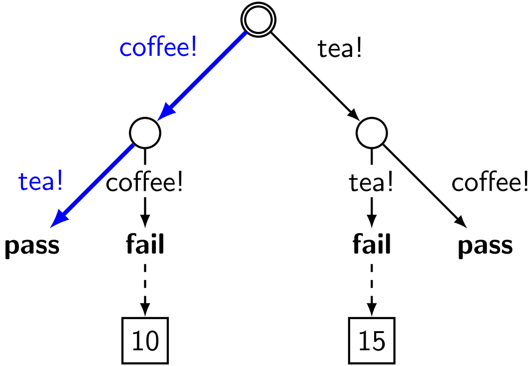

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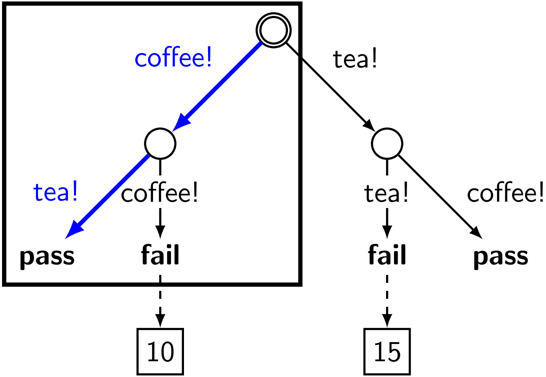

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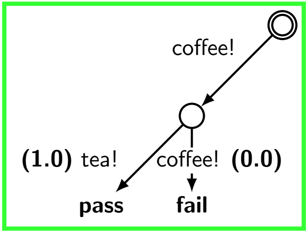

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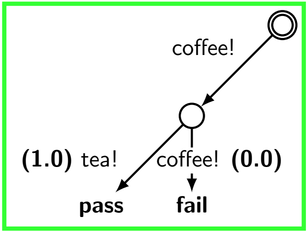

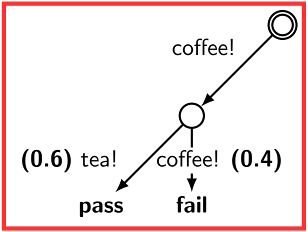

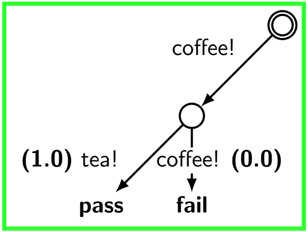
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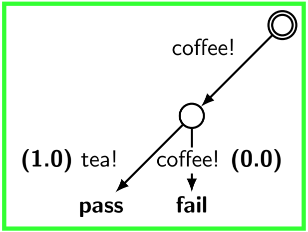
(1.0) tea!
coffee! (0.0)
pass
fail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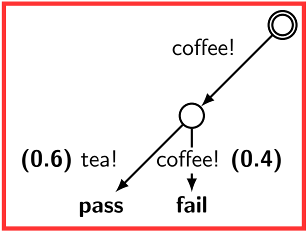
coffee!
(0.6) tea!
coffee! (0.4)
pass
fail

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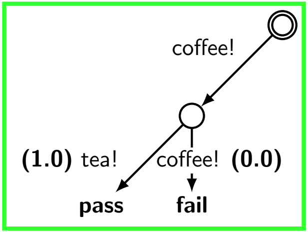

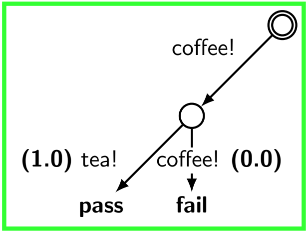

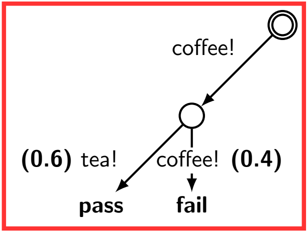

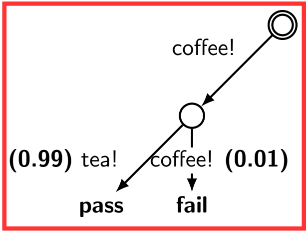

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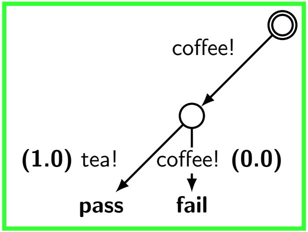

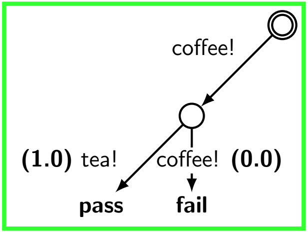

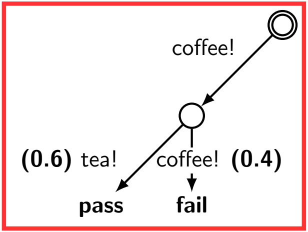

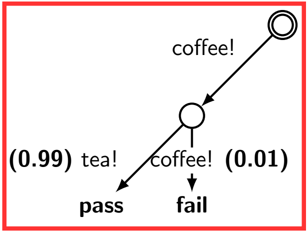

Conditional branching probabilities p^{cbr}

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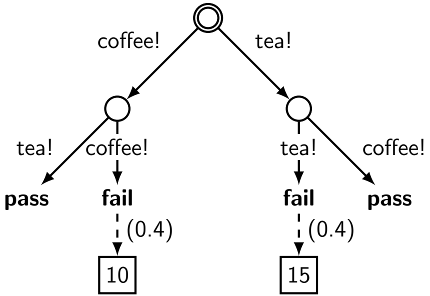

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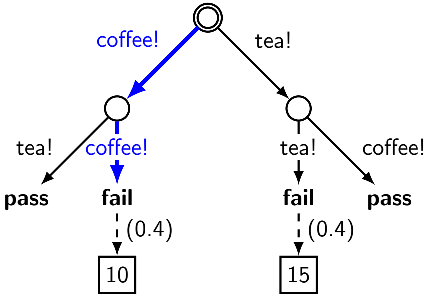

Fault coverage *coffee! coffee!*
10

Fault coverage *tea! tea!*
0

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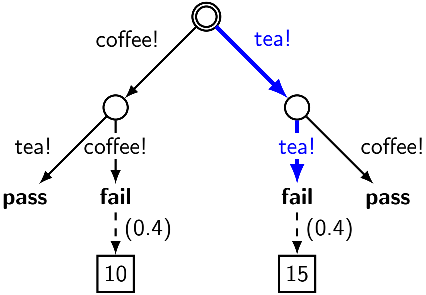

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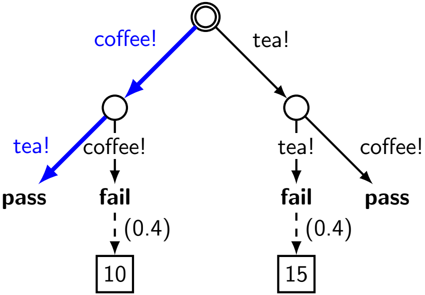

Fault coverage *coffee! coffee!*
4 – 6.4 – 7.8 – 8.7

Fault coverage *tea! tea!*
0

Evaluating actual coverage

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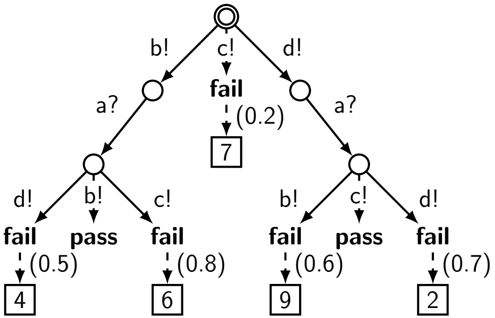
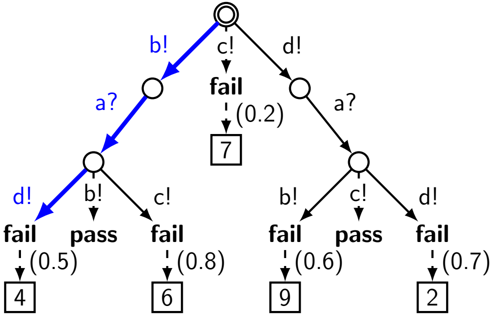
Actual coverage of an execution or sequence of executions:
The sum of all fault coverages



Actual coverage

Actual coverage of an execution or sequence of executions:
The sum of all fault coverages



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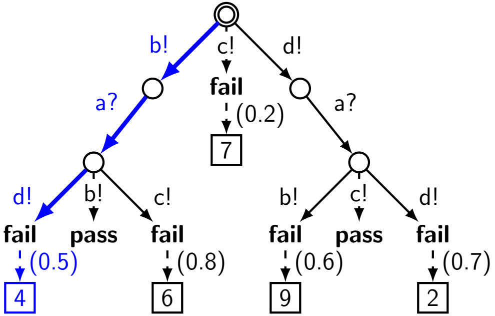
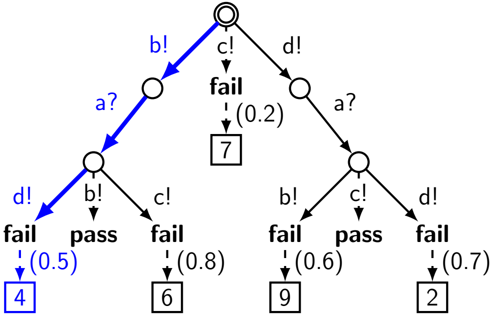


faultCov(b! a? d!) =

Actual coverage

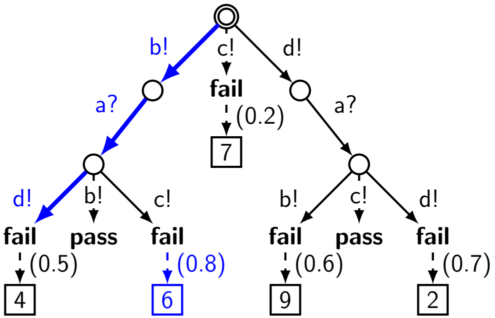
Actual coverage of an execution or sequence of executions:
The sum of all fault coverages



$$\mathit{faultCov}(b!\ a?\ d!) = 4$$

Actual coverage

Actual coverage of an execution or sequence of executions:
The sum of all fault coverages

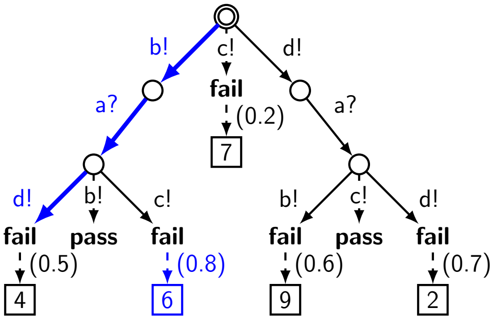


$$
\begin{aligned}
\mathit{faultCov}(b!\ a?\ d!) &= 4 \\
\mathit{faultCov}(b!\ a?\ c!) &=
\end{aligned}
$$

Evaluating actual coverage

Actual coverage

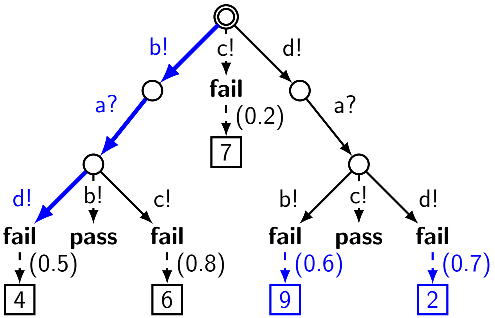
Actual coverage of an execution or sequence of executions:
The sum of all fault coverages



$$
\begin{aligned}
\mathit{faultCov}(b!\ a?\ d!) &= 4 \\
\mathit{faultCov}(b!\ a?\ c!) &= 4.8
\end{aligned}
$$

Actual coverage

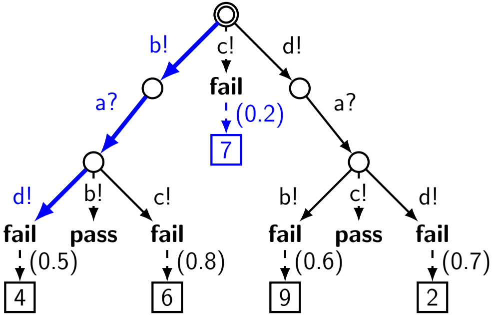
Actual coverage of an execution or sequence of executions:
The sum of all fault coverages



$$\begin{aligned} \mathit{faultCov}(b!\ a?\ d!) &= 4 \\ \mathit{faultCov}(b!\ a?\ c!) &= 4.8 \\ \mathit{faultCov}(d!\ a?\ b!) &= 0 \\ \mathit{faultCov}(d!\ a?\ d!) &= 0 \end{aligned}$$

Actual coverage

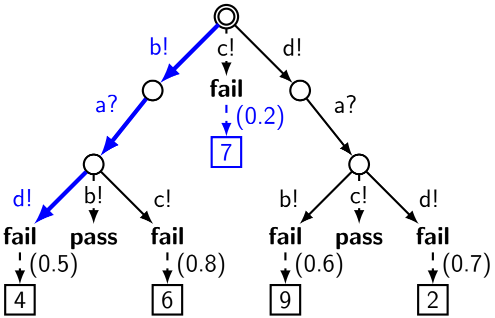
Actual coverage of an execution or sequence of executions:
The sum of all fault coverages



$$
\begin{aligned}
faultCov(b!\ a?\ d!) &= 4 \\
faultCov(b!\ a?\ c!) &= 4.8 \\
faultCov(d!\ a?\ b!) &= 0 \\
faultCov(d!\ a?\ d!) &= 0 \\
faultCov(c!) &=
\end{aligned}
$$

Actual coverage

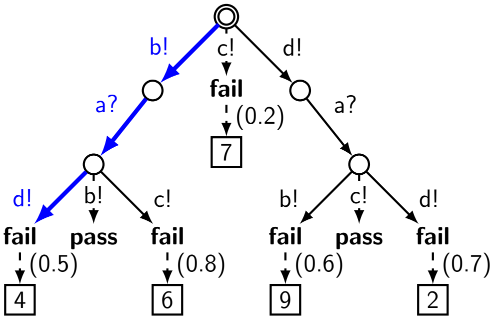
Actual coverage of an execution or sequence of executions:
The sum of all fault coverages



$$
\begin{aligned}
faultCov(b!\ a?\ d!) &= 4\\
faultCov(b!\ a?\ c!) &= 4.8\\
faultCov(d!\ a?\ b!) &= 0\\
faultCov(d!\ a?\ d!) &= 0\\
faultCov(c!) &= 1.4
\end{aligned}
$$

Actual coverage

Actual coverage of an execution or sequence of executions:
The sum of all fault coverages



$$
\begin{aligned}
faultCov(b!\ a?\ d!) &= 4\\
faultCov(b!\ a?\ c!) &= 4.8\\
faultCov(d!\ a?\ b!) &= 0\\
faultCov(d!\ a?\ d!) &= 0\\
faultCov(c!) &= 1.4\\
\\
absCov &= 10.2
\end{aligned}
$$

Predicting actual coverage

Actual coverage distribution of a test case

The *actual coverage distribution* of a test case predicts its actual coverage.

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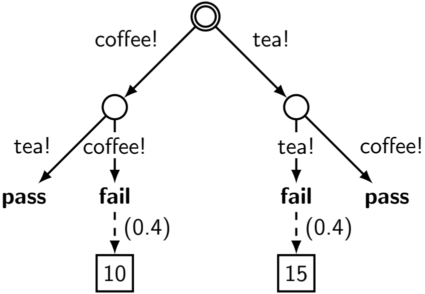

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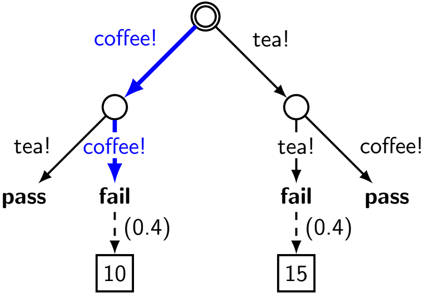
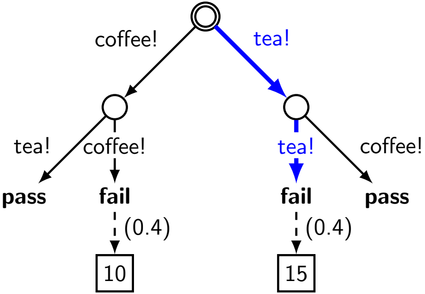


absCov
10

Predicting actual coverage

Actual coverage distribution of a test case

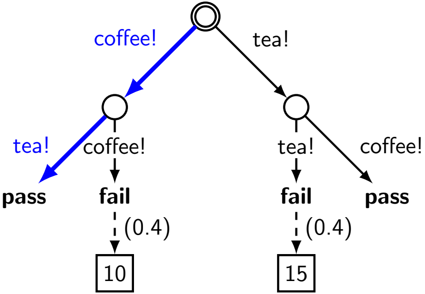
The *actual coverage distribution* of a test case predicts its actual coverage.



absCov
10
15

Actual coverage distribution of a test case

The *actual coverage distribution* of a test case predicts its actual coverage.

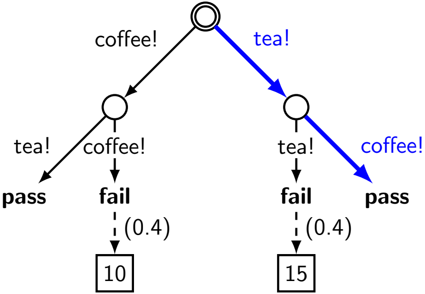


absCov
10
15
4

Predicting actual coverage

Actual coverage distribution of a test case

The *actual coverage distribution* of a test case predicts its actual coverage.

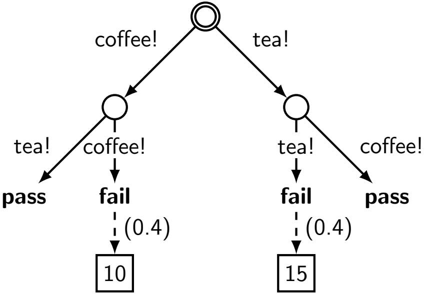


absCov
10
15
4
6

Predicting actual coverage

Actual coverage distribution of a test case

The *actual coverage distribution* of a test case predicts its actual coverage.

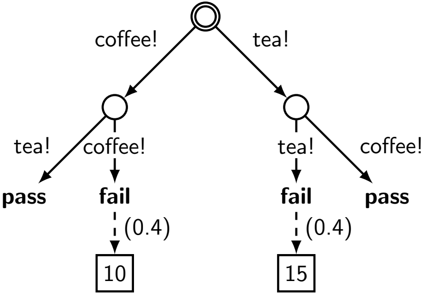


absCov	$\mathbb{P}$
10	0.015
15	0.005
4	0.735
6	0.245

Predicting actual coverage

Actual coverage distribution of a test case

The *actual coverage distribution* of a test case predicts its actual coverage.

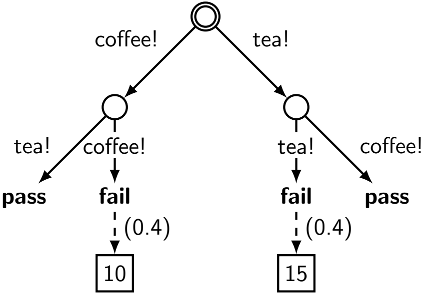


absCov	$\mathbb{P}$	$\times$
10	0.015	0.150
15	0.005	0.075
4	0.735	2.940
6	0.245	1.470

Predicting actual coverage

Actual coverage distribution of a test case

The *actual coverage distribution* of a test case predicts its actual coverage.

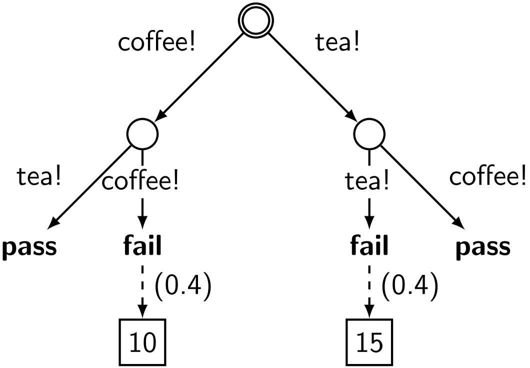


absCov	$\mathbb{P}$	$\times$
10	0.015	0.150
15	0.005	0.075
4	0.735	2.940
6	0.245	1.470 +

$$\mathbb{E}(absCov) = 4.635$$

Branching probabilities

The *branching probabilities* p^{br} describe how the implementation is expected to behave



Branching probabilities

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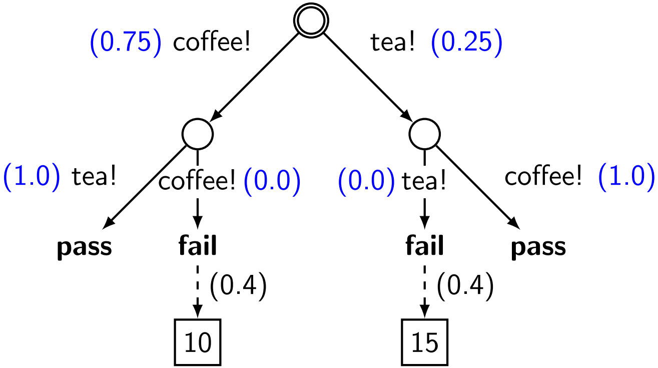
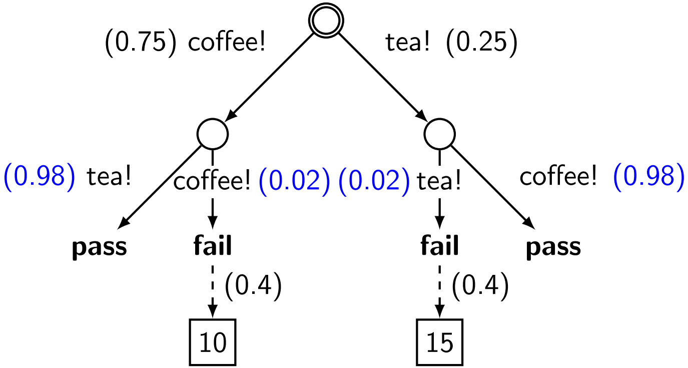


Branching probabilities

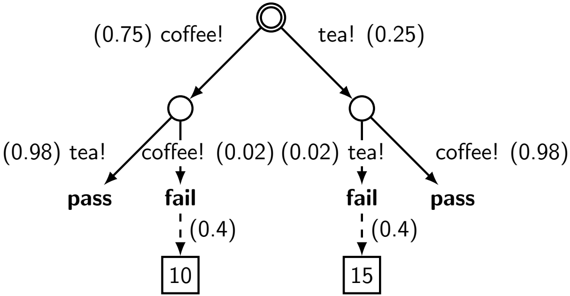
Branching probabilities

The *branching probabilities* p^{br} describe how the implementation is expected to behave



Sequences of executions

- Suppose we perform three executions of



Sequences of executions

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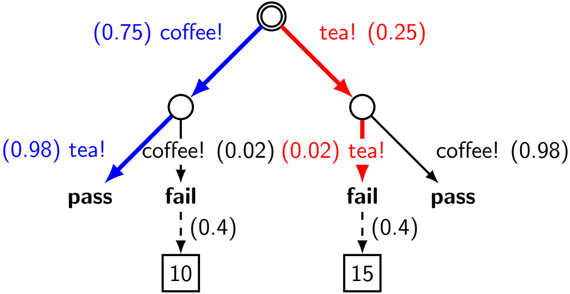
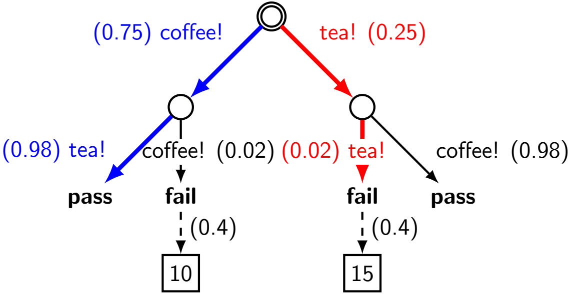


- Possible observation: [blue, blue, red]

Sequences of executions

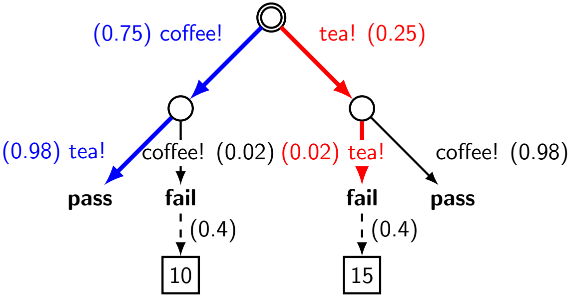
- Suppose we perform three executions of



- Possible observation: [blue, blue, red]
- Actual coverage: $15 + 6.4 = 21.4$

Sequences of executions

- Suppose we perform three executions of



- Possible observation: [blue, blue, red]
- Actual coverage: $15 + 6.4 = 21.4$
- Many observations possible: $O(|exec|^n)$

Expected actual coverage for a sequence of executions

Theorem

$$\mathbb{E}(actCov_n) = \sum_{\sigma a \in err_t} f(\sigma a) \cdot \Bigg((1 - (1 - p^{\mathrm{to}}(\sigma a))^n) \cdot 1 + \sum_{k=0}^{n} \binom{n}{k} p^{\mathrm{to}}(\sigma)^k (1 - p^{\mathrm{to}}(\sigma))^{n-k} \cdot (1 - p^{\mathrm{br}}(a \mid \sigma))^k \cdot (1 - (1 - p^{\mathrm{cbr}}(a \mid \sigma)^k)) \Bigg)$$

Expected actual coverage for a sequence of executions

Theorem

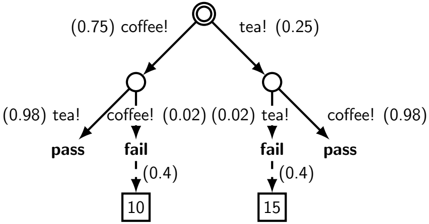
$$\mathbb{E}(actCov_n) = \sum_{\sigma a \in err_t} f(\sigma a) \cdot \Bigg((1-(1-p^{\mathrm{to}}(\sigma a))^n) \cdot 1 + \sum_{k=0}^{n} \binom{n}{k} p^{\mathrm{to}}(\sigma)^k (1-p^{\mathrm{to}}(\sigma))^{n-k} \cdot (1-p^{\mathrm{br}}(a \mid \sigma))^k \cdot (1-(1-p^{\mathrm{cbr}}(a \mid \sigma)^k)) \Bigg)$$

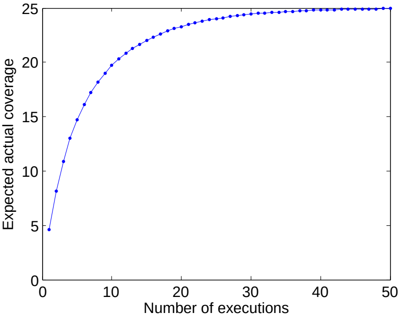
Expected actual coverage for a sequence of executions

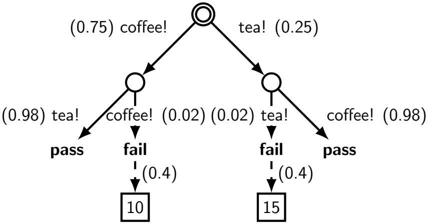
Theorem

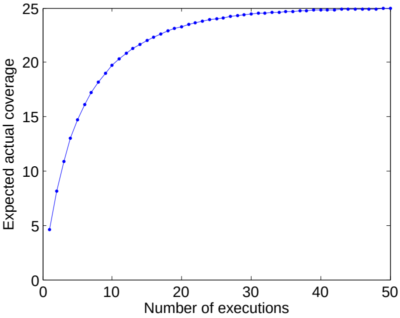
$$\mathbb{E}(actCov_n) = \sum_{\sigma a \in err_t} f(\sigma a) \cdot \Bigg((1 - (1 - p^{\mathrm{to}}(\sigma a))^n) \cdot 1 + \sum_{k=0}^{n} \binom{n}{k} p^{\mathrm{to}}(\sigma)^k (1 - p^{\mathrm{to}}(\sigma))^{n-k} \cdot (1 - p^{\mathrm{br}}(a \mid \sigma))^k \cdot (1 - (1 - p^{\mathrm{cbr}}(a \mid \sigma)^k)) \Bigg)$$

Asympotical behaviour of actual coverage









Theorem

$$\lim_{n\to\infty} \mathbb{E}(\text{actual coverage}_n) = \text{potential coverage}$$

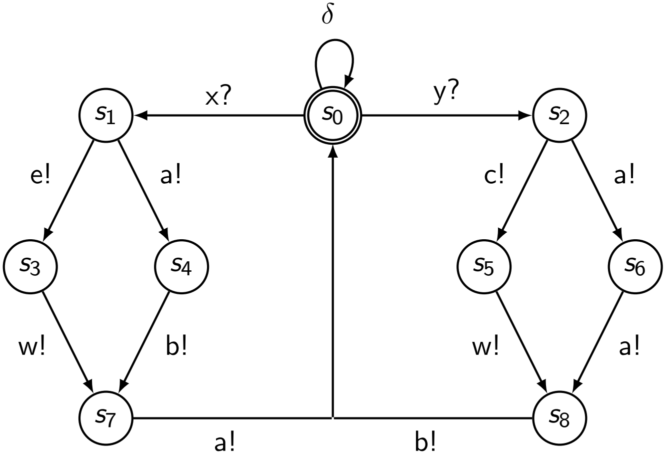
Actual coverage for test suites

- Very similar to actual coverage for test cases:
  sum all the fault coverages
- Take into account in how many test cases an erroneous trace is contained
- Again, an efficient formula for the expected actual coverage exists

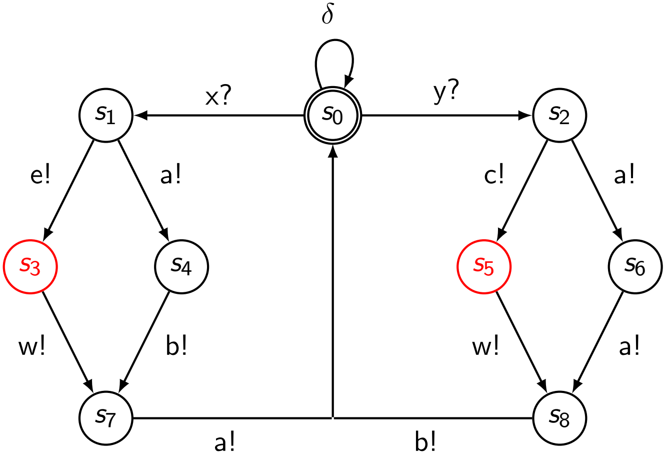
Theorem

$$\lim_{n \to \infty} \mathbb{E}(\text{actual coverage}_n) = \text{potential coverage}$$

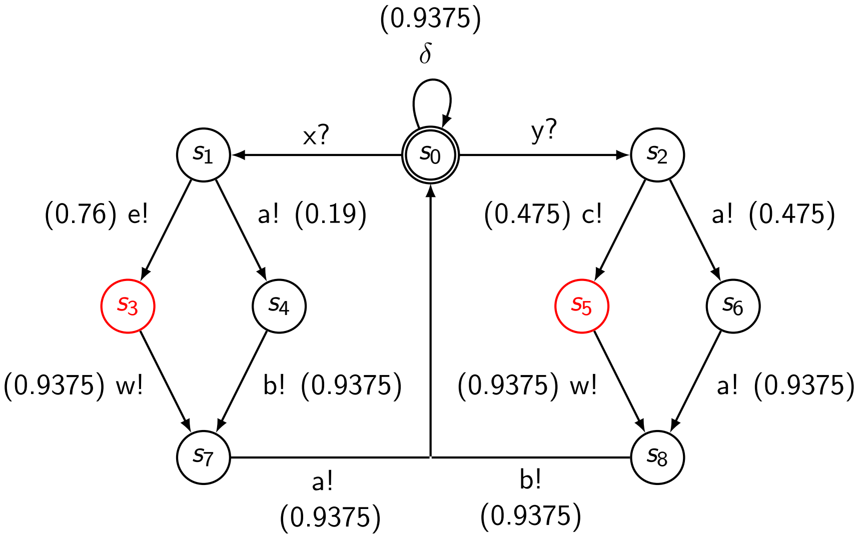
Example – chemical dispenser



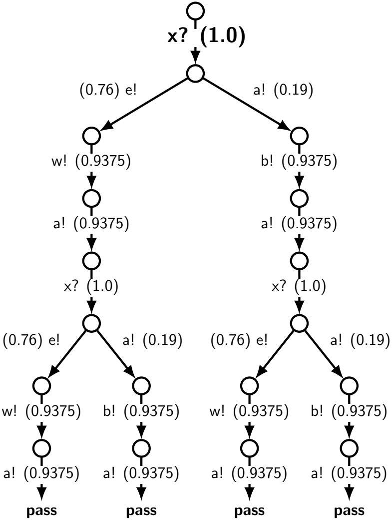
Example – chemical dispenser



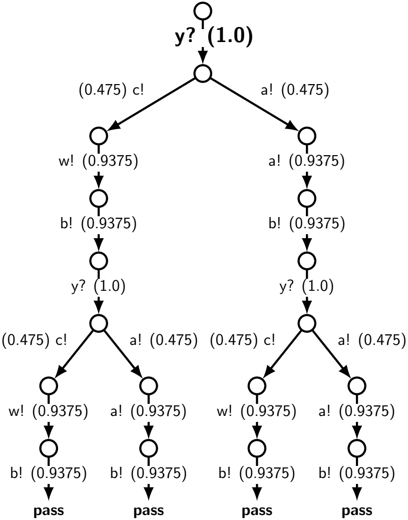
Example – chemical dispenser



Example – chemical dispenser



(a) Test case t_1 (pot.cov.: 1938)



(b) Test case t_2 (pot.cov.: 1938)

Prediction of actual coverage

$$
\begin{aligned}
\mathbb{E}(A^{1}_{t_1}) &= 197.0 \\
\mathbb{E}(A^{5}_{t_1}) &= 729.1 \\
\mathbb{E}(A^{10}_{t_1}) &= 1076.9 \\
\mathbb{E}(A^{50}_{t_1}) &= 1704.6 \\
\mathbb{E}(A^{250}_{t_1}) &= 1917.7
\end{aligned}
\qquad
\begin{aligned}
\mathbb{E}(A^{1}_{t_2}) &= 156.8 \\
\mathbb{E}(A^{5}_{t_2}) &= 639.8 \\
\mathbb{E}(A^{10}_{t_2}) &= 1032.1 \\
\mathbb{E}(A^{50}_{t_2}) &= 1848.0 \\
\mathbb{E}(A^{250}_{t_2}) &= 1938.0
\end{aligned}
$$

Prediction of actual coverage

$$
\begin{array}{lclclcl}
\mathbb{E}(A^{1}_{t_1}) & = & 197.0 & > & \mathbb{E}(A^{1}_{t_2}) & = & 156.8 \\
\mathbb{E}(A^{5}_{t_1}) & = & 729.1 & > & \mathbb{E}(A^{5}_{t_2}) & = & 639.8 \\
\mathbb{E}(A^{10}_{t_1}) & = & 1076.9 & > & \mathbb{E}(A^{10}_{t_2}) & = & 1032.1 \\
\mathbb{E}(A^{50}_{t_1}) & = & 1704.6 & < & \mathbb{E}(A^{50}_{t_2}) & = & 1848.0 \\
\mathbb{E}(A^{250}_{t_1}) & = & 1917.7 & < & \mathbb{E}(A^{250}_{t_2}) & = & 1938.0
\end{array}
$$

Simulation and evaluation actual coverage

	t_1			t_2		
n	$\mathbb{E}(A^n_{t_1})$	$\overline{\text{Sim.}}$	std.	$\mathbb{E}(A^n_{t_2})$	$\overline{\text{Sim.}}$	std.
1	197.0	213.3	50.1	156.8	155.1	60.8
5	729.1	762.1	84.0	639.8	629.7	107.0
10	1076.9	1112.6	104.8	1032.1	1013.3	114.8
50	1704.6	1743.3	62.4	1848.0	1831.2	39.5
250	1917.7	1925.8	11.2	1938.0	1938.0	0.0

	t_1			t_2		
n	$\mathbb{E}(A^n_{t_1})$	$\overline{\text{Sim.}}$	std.	$\mathbb{E}(A^n_{t_2})$	$\overline{\text{Sim.}}$	std.
1	197.0	229.1	48.4	156.8	174.7	52.0
5	729.1	813.9	56.5	639.8	711.6	89.4
10	1076.9	1167.9	94.8	1032.1	1133.6	92.9
50	1704.6	1757.3	62.9	1848.0	1890.7	19.4
250	1917.7	1926.0	11.1	1938.0	1938.0	0.0

Main results

- New notion of coverage: *actual coverage*
- Evaluating actual coverage of a given execution
- Predicting actual coverage of a test case or test suite

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For more details, see my Master's Thesis
(`wwwhome.cs.utwente.nl/~timmer`)

Conclusions and future work

Main results

- New notion of coverage: *actual coverage*
- Evaluating actual coverage of a given execution
- Predicting actual coverage of a test case or test suite

For more details, see my Master's Thesis
(`wwwhome.cs.utwente.nl/~timmer`)

Directions for future work

- Validation of the framework: tool support, case studies
- Dependencies between errors
- Accuracy of approximations
- On-the-fly test derivation

Questions